

Unit 5: Solving Quadratic Equations ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Solving Quadratic Equations by Factoring

1. Determine algebraically whether the given value is a solution for each quadratic equation.

a) $2x^2 - 5x - 3 = 0$, $x = 3$

b) $3x^2 - 9 = 0$, $x = \sqrt{6}$

2. Solve each quadratic equation.

a) $(x + 3)(x + 4) = 0$

b) $(x + 5)(3x - 4) = 0$

c) $2(x + 4) = 0$

d) $3x(x - 8) = 0$

3. Solve by factoring.

a) $m^2 - 12m + 35 = 0$

b) $4c^2 - 49 = 0$

c) $y^2 - 11y = 0$

d) $2x^2 - 7x - 15 = 0$

4. Solve by expanding, grouping like terms, and then factoring.

a) $m^2 + 8m = 3m + 24$

b) $x(x - 6) = 2(x - 8)$

c) $(2x - 1)(x - 3) = (x + 1)(x - 2)$

d) $(2p - 1)^2 - 3 = (p - 2)(p - 1)$

5. Write an expanded quadratic equation, with integer coefficients, from the given roots.

a) $x = -5, -3$

b) $x = \frac{1}{3}, \frac{2}{3}$

c) $x = 4, -\frac{1}{2}$

d) $x = 0, \frac{4}{3}$

6. Solve by factoring.

a) $x = \sqrt{3x - 2}$

b) $\sqrt{31 - x} = x - 1$

7. Find algebraically two consecutive integers with a product of 156.

8. Determine algebraically what number and its square differ by 30?
9. The hypotenuse of a right triangle is 29cm . If the other two sides differ by 1cm , what are their lengths?
10. When a football is kicked with a vertical speed of 20m/s , its height, h meters, after t seconds is approximated by the formula: $h = 20t - 5t^2$. How long after the kick is the football at a height of 15m ?

11. A rectangular garden has dimensions $6m$ by $8m$. Leonard wants to build a path around the garden. He has $120m^2$ of bricks. How wide is the pathway?

12. Solve by factoring.

a) $(2x + 1)^2 - 3(2x + 1) - 4 = 0$

b) $3(x - 1)^2 - 8(x - 1) + 5 = 0$

13. If one root is 5, find the value of k as well as the other root.

a) $x^2 - 3x + k = 0$

b) $x^2 + kx + 40 = 0$

Lesson Two

Solving Quadratics by the Square Root Principle

1. Solve each equation exactly.

a) $x^2 + 12 = 48$

b) $2x^2 - 32 = 0$

c) $m^2 - 7.5 = 92.5$

d) $3x^2 + 2 = 29$

e) $10 - z^2 = 4$

f) $-3 = 12 - 5d^2$

g) $(y + 2)^2 = 9$

h) $(3 - c)^2 - 16 = 0$

i) $\left(m - \frac{1}{3}\right)^2 = \frac{4}{9}$

j) $(1 - 2g)^2 = 49$

k) $1.21 = (10a + 1)^2$

l) $\frac{1}{3}(6x - 1)^2 - 3 = 0$

m) $\frac{(a+2)^2}{4} = 16$

n) $\left(a + \frac{3}{4}\right)^2 = \frac{3}{16}$

o) $ax^2 - b = 0$

2. The height, h meters, of a falling object is related to the time, t seconds, the object has been falling by the formula $h = -4.9t^2 + d$ where d meters is the initial height of the object above the ground. The Bankers Hall building in Calgary is $196m$ tall. Express the time an object takes to reach the ground from this height as an exact number of seconds.
3. The volume of a cone with height, h (m), and radius r (m), is given by the formula $v = \frac{1}{3}\pi r^2 h$. What is the radius of a cone with volume $168cm^3$ and height $9cm$.
4. Five is added to an integer that is being quartered and then squared. This sum equals nine. Determine the integer algebraically.

5. The surface area of a cube is 120cm^2 . Determine the exact length of one of the edges.

6. Thirty less than the square of an integer is 139. Find the integer algebraically.

7. Solve each equation exactly.

a) $\frac{1}{x^2} = 16$

b) $\frac{1}{x^2-1} = 5$

c) $\frac{7}{x+3} = x - 3$

d) $\frac{4}{(x-1)^2} = \frac{9}{4}$

Lesson Three

Solving Quadratics Using the Quadratic Formula

1. Solve using the quadratic formula. Give exact answers as integers, fractions, or simplest radical form.

a) $2x^2 - 5x + 2 = 0$

b) $3n^2 - 11n - 14 = 0$

c) $f^2 - 6f + 4 = 0$

d) $x^2 + 7x + 3 = 0$

e) $6t^2 - t - 1 = 0$

f) $5h^2 + 7h + 2 = 0$

2. Solve exactly using the quadratic formula.

a) $25q^2 + 70q + 49 = 0$

b) $12v^2 - 192 = 0$

c) $3r^2 - 4r = 0$

d) $5x^2 + 12x + 9 = 0$

3. Solve using the quadratic formula. Express answers to the nearest hundredth.

a) $0.2z^2 - z - 3.2 = 0$

b) $1.2x^2 = 1.4x - 1$

4. Solve using the quadratic formula. Answer in simplest radical form.

a) $2x^2 + 6x + 2 = 0$

b) $z^2 - 6z + 7 = 0$

5. Solve using the quadratic formula.

a) $(n - 4)(n - 2) = 12$

b) $(2x - 1)(3x + 5) = (x + 2)(2x - 1)$

c) $3m^2 - (5m + 1)(2m - 3) = 3$

d) $2(w - 2)(w + 1) - (w + 3) = 0$

6. A photograph measuring 16cm by 12cm is to be surrounded by a mat before framing. The width of the mat is to be the same on all sides of the photograph. The area of the mat is equal to the area of the photograph. Find the width of the mat, to the nearest tenth of a centimeter.
7. The hypotenuse of a right triangle measures 20cm . The sum of the lengths of the other two sides is 28cm . Find the lengths of these two sides.

8. Elli travelled 700km by truck to deliver some ostriches to her friend's farm, she then returned the same distance by car. The car averaged 10km/h faster than the truck and took 1 hour less time. What was the speed of the car?

9. When two pumps are turned on they can fill the pond at Helmut's farm in seven hours. Individually, one pump takes two hours longer than the other to fill the pond. Find the time to fill the pond by the slower pump by itself.

10. A boat travelled 30km upstream, against the current, to deliver some feed from Elli, to Helmut's farm, it then turned around and headed 35km downstream to dock for the night. If the speed of the current was 3km/h and the upstream trip took 45min longer than the downstream trip, what was the speed of the boat?

Lesson Four

The Discriminant

1. Calculate the value of the discriminant and then state the nature of the roots.

a) $x^2 + 11x + 24 = 0$

b) $n^2 - 4n + 2 = 0$

c) $4m^2 - 20m + 25 = 0$

d) $2x^2 - 5x + 8 = 0$

e) $\sqrt{5}x^2 + 7x + 2\sqrt{5} = 0$

f) $\frac{x-1}{2} - x^2 = 3$

2. Determine the value(s) of k that give the type of solution indicated.

a) $x^2 - 6x + k = 0$; one real root

b) $kx^2 - 2x + 1 = 0$; two real roots

c) $x^2 + 4x - 2k = 0$; no real roots

d) $2kx^2 - 4x + 3 = 0$; two real roots

e) $m^2 + 4km + 1 = 0$; one real root

f) $(k + 1)x^2 - 2x - 3 = 0$; no real roots

ANSWERS**Lesson One Solving Quadratics by Factoring**

1. a) Yes b) No 2. a) $x = -3, -4$ b) $x = -5, \frac{4}{3}$ c) $x = -4$ d) $x = 0, 8$
3. a) $m = 5, 7$ b) $c = \pm \frac{7}{2}$ c) $y = 0, 11$ d) $x = -\frac{3}{2}, 5$
4. a) $m = -8, 3$ b) $x = 4$ c) $x = 1, 5$ d) $p = -1, \frac{4}{3}$
5. a) $x^2 + 8x + 15 = 0$ b) $9x^2 - 9x + 2 = 0$ c) $2x^2 - 7x - 4 = 0$ d) $3x^2 - 4x = 0$
6. a) $x = 1, 2$ b) $x = 6$ 7. 12, 13 or $-12, -13$ 8. 6 or -5
9. 20cm and 21cm 10. 1 sec and 3 sec 11. 3m wide
12. a) $x = -1, \frac{3}{2}$ b) $x = \frac{8}{3}, 2$
13. a) $k = -10; x = -2$ b) $k = -13; x = 8$

Lesson Two Solving Quadratic by the Square Root Principle

1. a) $x = \pm 6$ b) $x = \pm 4$ c) $m = \pm 10$ d) $x = \pm 3$ e) $z = \pm\sqrt{6}$ f) $d = \pm\sqrt{3}$
- g) $y = -5, 1$ h) $c = -1, 7$ i) $m = -\frac{1}{3}, 1$ j) $g = -3, 4$ k) $a = -0.21, 0.01$
- l) $x = \frac{2}{3}, -\frac{1}{3}$ m) $a = -10, 6$ n) $a = \frac{\pm\sqrt{3}-3}{4}$ o) $x = \pm\sqrt{\frac{b}{a}}$
2. $t = \sqrt{40} \text{ sec}$ 3. radius = 4.2m 4. ± 8 5. edge = $2\sqrt{5} \text{ cm}$ 6. ± 13
7. a) $x = \pm \frac{1}{4}$ b) $x = \pm\sqrt{\frac{6}{5}}$ c) $x = \pm 4$ d) $x = \pm \frac{4}{3} + 1$

Lesson Three Solving Quadratics Using the Quadratic Formula

1. a) $x = \frac{1}{2}, 2$ b) $n = -1, \frac{14}{3}$ c) $f = 3 \pm \sqrt{5}$ d) $x = \frac{-7 \pm \sqrt{37}}{2}$ e) $t = -\frac{1}{3}, \frac{1}{2}$ f) $h = -1, -\frac{2}{5}$
2. a) $q = -\frac{7}{5}$ b) $v = \pm 4$ c) $r = 0, \frac{4}{3}$ d) no real roots
3. a) $z = -2.12, 7.22$ b) no real roots 4. a) $x = \frac{-3 \pm \sqrt{5}}{2}$ b) $z = 3 \pm \sqrt{2}$
5. a) $n = \frac{6 \pm \sqrt{52}}{2}$ b) $x = -\frac{3}{2}, \frac{1}{2}$ c) $m = 0, \frac{13}{7}$ d) $w = \frac{3 \pm \sqrt{65}}{4}$
6. 2.85cm 7. 16cm by 12cm 8. 88.8km/h 9. 13.07hrs 10. 13.40km/h

Lesson Four The Discriminant

1. a) Two Real Roots b) Two Real Roots c) One Real Root d) No Real Roots
- e) Two Real roots f) No Real Roots
2. a) $k = 9$ b) $k < 1$ c) $k < -2$ d) $k < \frac{2}{3}$ e) $k = \pm \frac{1}{2}$ f) $k < -\frac{4}{3}$